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Hierarchical Modular Systems with Elements of Folk Art

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Abstract. Hierarchical modular systems are composed of identical elements assembled by systematic rules based on rotational group. The systems are characterized by hierarchical symmetries and their regular outlines defined by the last rules. To introduce fundamental an element of Japanese folk art into an initial module of our system, we can form various hierarchical symmetric patterns composed of identical elements. Since outline of a pattern is defined by a last rule, it can be mapped into regular or semi-regular tessellation and polyhedron. We show various examples from traditional uniform patterns to self-similar patterns.

1. Introduction

Presently it takes too much time and cost to construct large space systems such as the International Space Station. Sometimes we must modify or alter the mission and configuration of such systems as required to comply not only with technical but also social or political affairs. Therefore basic concepts for future large space systems need to be able to adjust to such various changes. We propose a concept of hierarchical modular systems as one of such basic concepts. The proposed systems consist of a number of identical modules, which are hierarchically assembled. We can compose arbitrary structures with various sizes and shapes to fit mission requirements.

Geometrical properties of our systems include the following three points: hierarchical symmetries and gaps, extendibility of shapes and sizes, and overall shapes defined by last mapping. Because of there three properties, we get the idea to introduce elements of folk art into our hierarchical modular systems in order to create new design of tiling or tessellation. All designed patterns are composed of identical elements of folk art with hierarchical symmetries. In this paper what is a hierarchical modular system and how to introduce elements of folk art by means of some examples.

2. Hierarchical Modular Systems and Their Mathematical Expression

For future space systems, we proposed hierarchical modular structure systems inspired

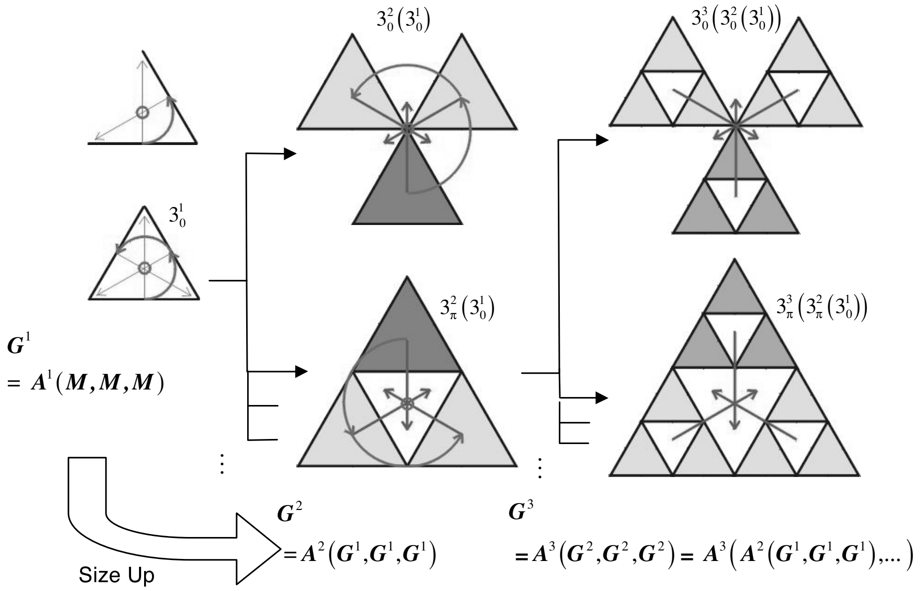


Fig. 1. Examples of configuration of 2-dimensional hierarchical modular structures.

by hierarchical property of natural things including fractal geometry (KISHIMOTO and NATORI, 2006). The proposed systems consist of a number of same-shaped modules, which are hierarchically assembled. They can form various sizes and shapes with the same-shaped modules connected by systematic assembly rules. The rules are expressed by the following mathematical equations.

$$
\begin{aligned}
G^1 &= A^1(M, M, \dots, M),\\
G^2 &= A^2\left(G^1, G^1, \dots, G^1\right),\\
&\vdots\\
G^k &= A^k\left(G^{k-1}, G^{k-1}, \dots, G^{k-1}\right)\\
&= A^k\left(A^{k-1}\left(G^{k-2}, \dots, G^{k-2}\right), \dots, A^{k-1}\left(G^{k-2}, \dots, G^{k-2}\right)\right)\\
&= \cdots,
\end{aligned}
\tag{1}
$$

where M is an initial member, A^k is k-th assembly rule to generate a next-generation structure, and G^k is a k-th generation structure. Assembly rule expressed by A^k may be different for each k. This expression shows that one G^k is composed of some previous generation G^{k-1}s. In particular, a uniform A^k generates the structures with fractal properties.

Assembly rules based on geometrical symmetry have been introduced because of

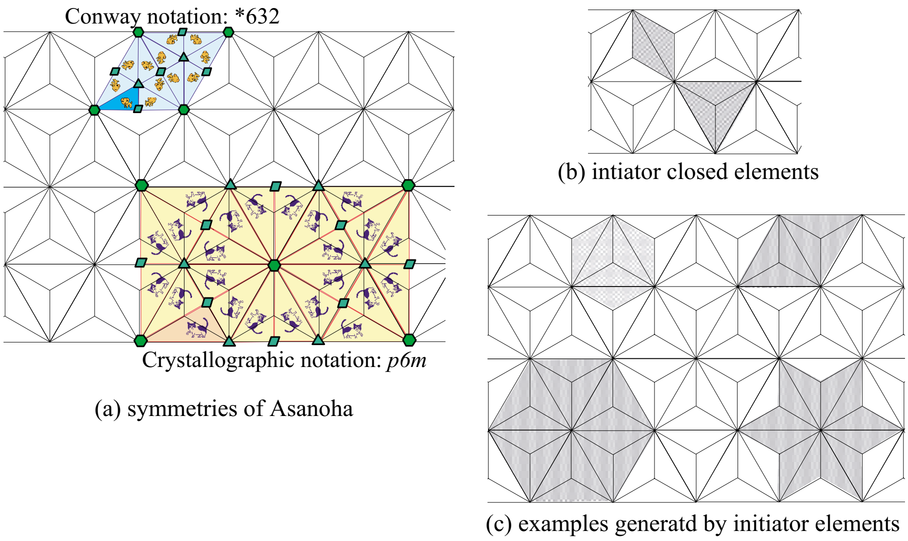


Fig. 2. Geometrical symmetries and selected initiator elements of Asanoha.

structural total balance. Especially we have proposed assembly rules based on rotation groups. According to this concept, two dimensional hierarchical modular structures are constructed according to two dimensional rotation groups (KISHIMOTO, 2004). Representative cases of first generations are all polyhedrons. In this case, an initial member M is one dimensional element. Using rotation mapping, a closed loop system such as polyhedron is generated. A mapping is characterized by a rotation with $2\pi/n$ [rad] around a center of a symmetric axis. In the same way, second generations are various closed loop systems made of polyhedrons. In the viewpoint of structure engineering, stiffness property of a closed loop system is rather higher. It is easy to construct arbitrary generation closed-loop shapes.

3. Hierarchical Modular Systems with Elements of Japanese Folk Art

3.1. Initiator elements based on geometrical symmetry

In this paper, we apply various initiator elements from folk art into a first module. An "initiator" element means a fundamental minimum element. Initiator elements of folk art are selected based on geometrical symmetry in order to manage connectivity between modules or module groups. Japanese traditional folk art Asanoha, which means flax ornament, is treated for purpose of illustration. Geometrical symmetries of Asanoha are shown in Fig. 2(a) described as *632 in Conway notation (CONWAY and HUSON, 2002) or p6m in crystallographic notation (IUCr, 2006). Figure 2(b) shows two initiator closed elements based on geometrical symmetries. Other elements shown in Fig. 2(c) might be selected, but they can be generated from these two initiator closed elements through

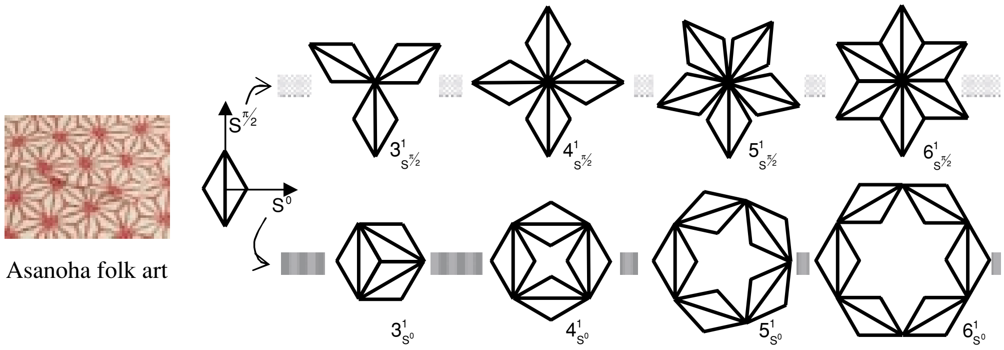


Fig. 3. Various first generations generated from an initiator element of Asanoha.

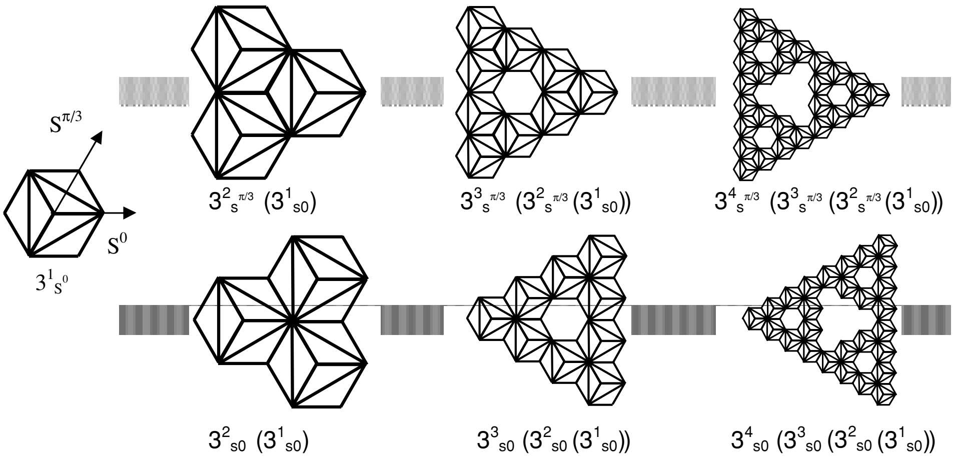


Fig. 4. Self similar composition generated from the first generations of Asanoha $3^1{}_S{}^0$.

rotational operation. In the same way, various initiator closed elements are selected as first generation module from many other planer patterns including traditional folk art.

3.2. Introduction of initiator elements from folk art into hierarchical modular systems

In this paper, we introduce of various unit elements from folk art into initiator modules. This attempt can be said as one of tiling or tessellation. However, conventional tiling and tessellation provide spatially uniform repeated patterns. In hierarchical modular systems, each assembly rule can be different every generation, which increase spatial diversity.

Various Japanese traditional patterns such as Asanoha, Kagome, Kikko, Shippo,

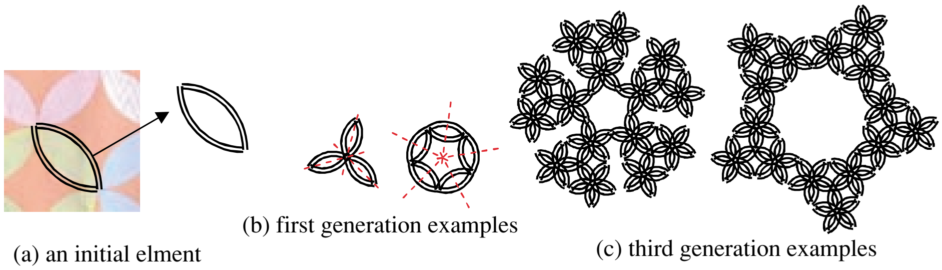


Fig. 5. Japanese traditional "shippo" and hierarchical modular system with its elements.

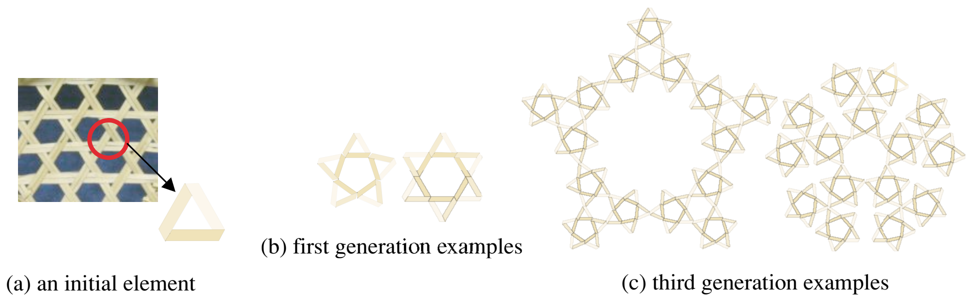


Fig. 6. Japanese traditional "kagome" and hierarchical modular system with its elements.

Sayagata, and so on, are available, however Asanoha is mainly treated in the following discussion. Figure 3 shows Asanoha, a selected initiator, and first generation examples. For each selected initiator, we can generate various first generations using several symmetric axes of an initiator and arbitrary two dimensional rotation mappings. Obviously a different symmetric axis or rotation mapping will generate a different pattern shown in Fig. 3. In the same way, we can extend various symmetric patterns of various generations. Figure 4 demonstrates self-similar examples based on a first generation ($3^1{}_S{}^0$) in Fig. 3. They are all different from each other, despite that all patterns are made of the identical initiator element and first generation modules. They also show hierarchical geometric symmetry. Each pattern is formed from some module (groups) with gaps, and its outline of total shape reflects its last mapping.

3.3. Examples based on various Japanese folk art

In the same way, any other various elements of Japanese folk art are available. Some examples based on other Japanese folk art than Asanoha are shown. Figures 5 and 6 are based on Japanese folk art Shippo and Kagome, respectively. In both examples, we can select an initial element considering geometrical symmetry (Figs. 5(a) and 6(a)),generate a first generation according to a 2-D rotational mapping generator (Figs. 5(b) and 6(b)), and

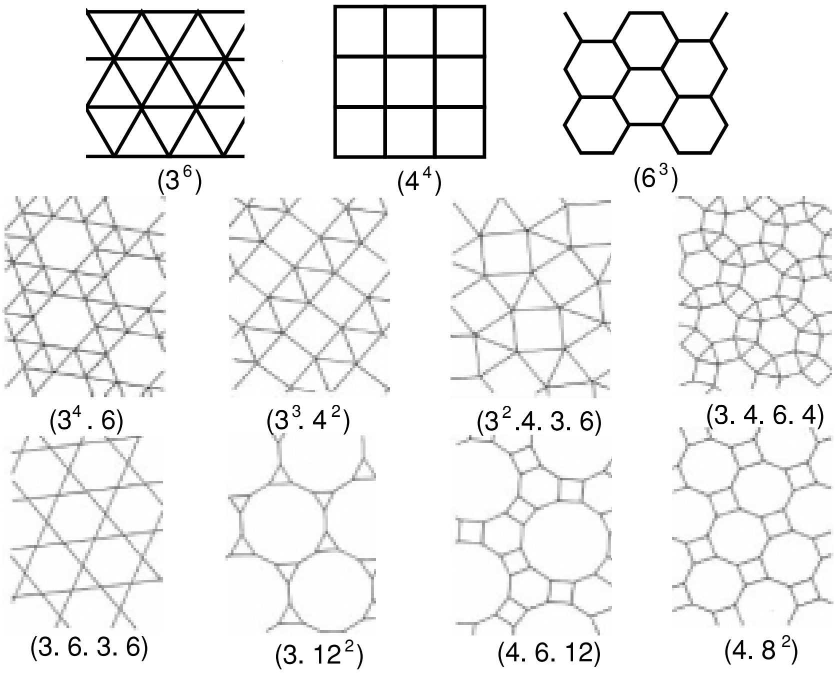


Fig. 7. Regular and semi-regular tessellation patterns

compose arbitrary generation patterns. Third generation examples are shown in Figs. 5(c) and 6(c). Obviously, we can select various initial elements, generate various first generations, and compose arbitrary generations other than as shown in these examples. Traditional uniform patterns can be generated by selecting appropriate initial elements and mapping generators.

4. Layout on Regular/Semi-regular Tessellation and Regular/Semi-regular Polyhedron

4.1. Geometrical properties of hierarchical modular systems

Hierarchical modular systems have some typical geometrical properties due to their composition procedures. They have hierarchical geometrical symmetry, hierarchically arranged gaps, and extendibility. There is one more important property that an outline of a hierarchical modular system is defined the last mapping. This property allows us to introduce our hierarchical modular systems into regular/semi-regular tessellation. Figure 7 shows examples of regular and semi-regular tessellation patterns. Regular tessellation patterns are composed of only one kind of regular polygons, and semi-regular ones are composed of several kinds of regular polygons. We can easily replace each polygon by a pattern of hierarchical modular systems, because of the outline of the pattern defined by the last mapping.

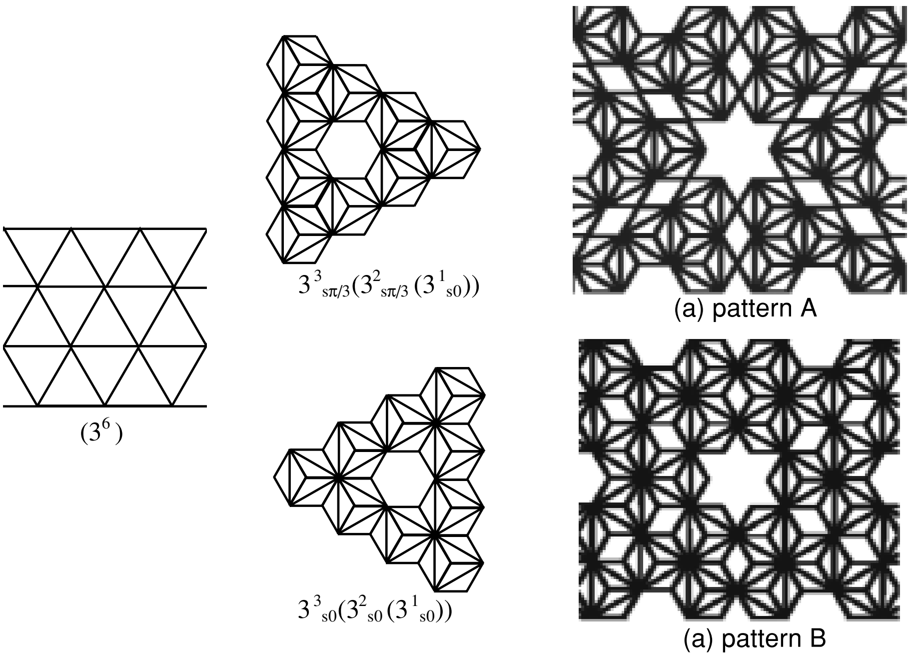


Fig. 8. Two different pattern examples generated by regular tessellation 3^6.

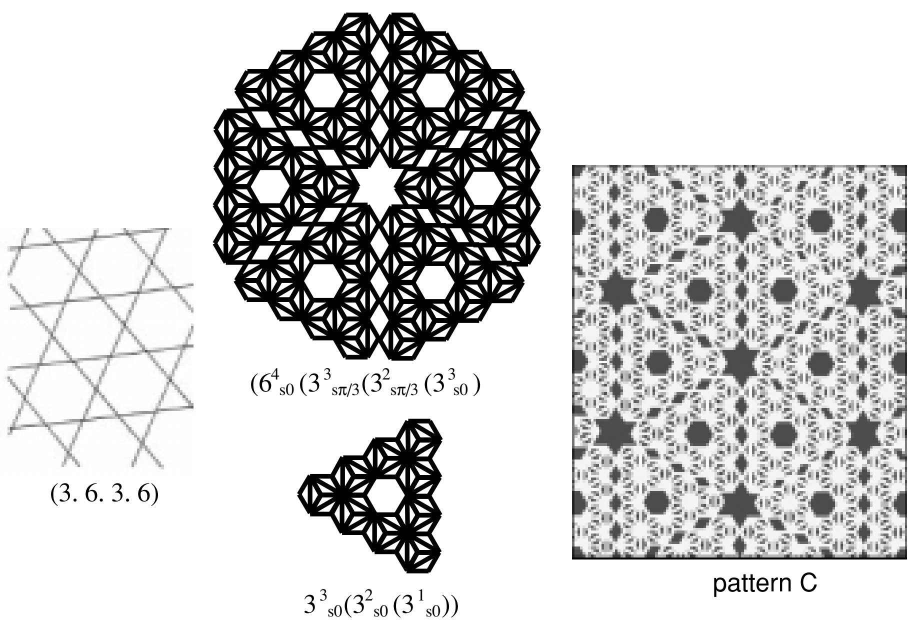


Fig. 9. Pattern example generated by semi-regular tessellation (3.6.3.6).

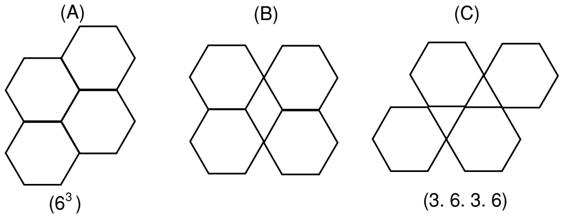


Fig. 10. Three tessellation patterns based on regular hexagon.

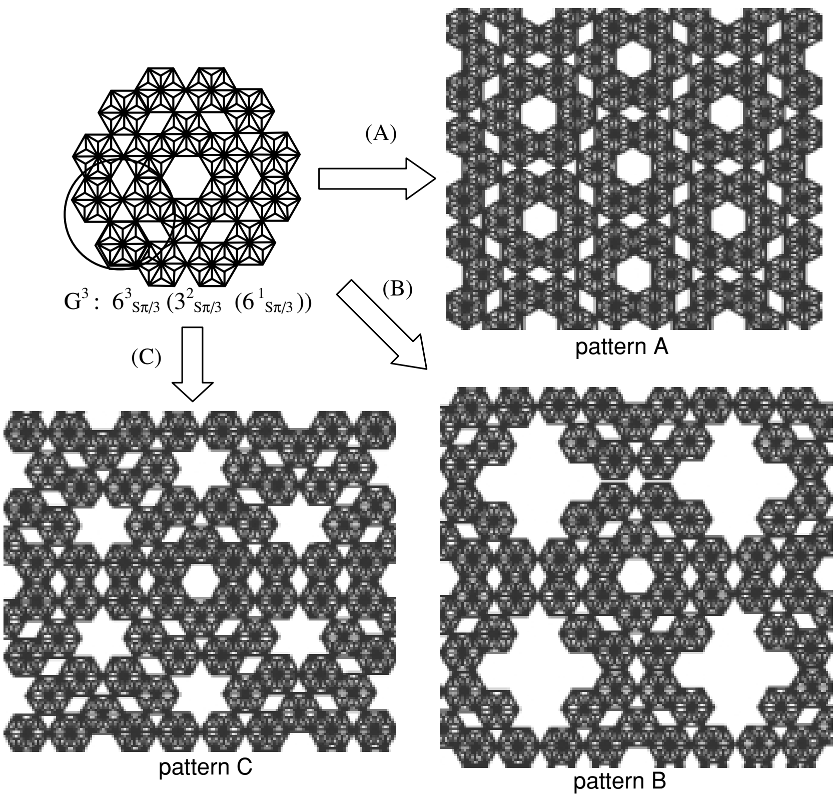
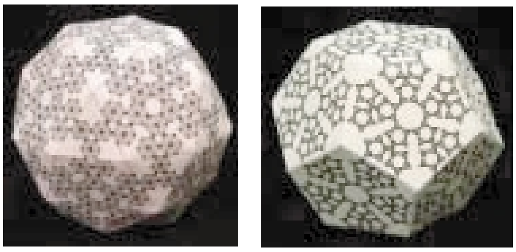


Fig. 11. Pattern example generated by tessellation with regular and no-regular polygons.

4.2. Layout on regular/semi-regular tessellation and tiling

Figures 8 and 9 are examples introducing third generation in Fig. 4 into regular tessellation 3^6 and semi-regular tessellation (3.6.3.6), respectively. In semi-regular tessellation case, third generation $3^3_{s0}\ (3^2_{s0}\ (3^1_{s0}))$ and fourth generation $6^4_{s0}\ (3^3_{s\pi/3} \times (3^2_{s\pi/3}\,(3^1_{s0})))$ are used. A third generation included in $6^4_{s0}\,(3^3_{s\pi/3}\,(3^2_{s\pi/3}\,(3^1_{s0})))$is different



(a) icosidodecahedron (b) dodecahedron

Fig. 12. Layout examples on regular and semi-regular polyhedron. (a) Icosidodecahedron. (b) Dodecahedron.

from $3^3{}_{s0}\,(3^2{}_{s0}\,(3^1{}_{s0}))$. At first glance it seems to be uniform, but a second look confirms that there are two different third generation patterns in it. These examples show hierarchical modular systems can provide diversified patterns from elements of folk art.

However, Figs. 8 and 9 treat tessellation with only regular polygons, tessellations including no-regular polygons are also available. For simplicity, Fig. 10 shows considered three tessellation patterns based on regular hexagons. Figures 10(a) and (c) are the case with only regular hexagon and triangle. Figure 10(b) is the case with regular hexagon and rhomboid. As shown in Fig. 11, a third generation based on Asanoha with a hexagonal counter is applied into these three tessellation patterns. Using identical generations, we can provide diversified design.

4.3. Layout on regular/semi-regular polyhedron

Accordingly, it is easy to lay out generated patterns on development elevation of regular or semi-regular polyhedron. Figures 12(a) and (b) illustrate examples of a third generation of Asanoha on an icosidodecahedron, and a third generation of Kagome on a dodecahedron, respectively.

5. Conclusions

This paper described how to introduce elements of Japanese folk art into hierarchical modular systems. Hierarchical modular systems are composed of identical elements assembled by systematic rules based on rotational group. Many of Japanese folk art has uniform repeated patterns. According to the concept of hierarchical modular systems, a selected initiator element can provide various patterns using various 2-D rotational mappings. We also be able to easily introduce our hierarchical modular systems into conventional tessellation, because outlines of hierarchical modular systems are defined by the last mappings. In this paper various examples with elements of folk art are demonstrated.

REFERENCES

CONWAY, J. H. and HUSON, D. H. (2002) The orbifold notation for two-dimensional groups, *Structural Chemistry*, **13**, 247–256.

IUCr (ed.) (2006) *International Tables for Crystallography*, Springer.

KISHIMOTO, N. (2004) Research on structure systems based on hierarchical concepts, Ph.D. Thesis of Tokyo University.
KISHIMOTO, N. and NATORI, M. C. (2006) Hierarchical modular structures and their geometrical configurations for future large space structures, *Journal of International Association of Sell and Spatial Structures*, December issue, 303–309.